Are we limited to a mere relative reality?

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Geometry and Experience

The physicist model of nature \((N)\) consists of a geometry \((G)\) and a purport \((P)\) of physical laws and only the sum \((G) + (P)\) is subject to the control of experience. The idea is Poincaré’s and discussed by Einstein in his 1921 talk “Geometry and Experience”.

Today, we model nature \((N_r)\) as \((G_r) + (P_r)\). We may not be limited to a relative reality if an alternative model \((N_a) = (G_a) + (P_a)\) passes the tests for \((N_r)\). Can a \((P_a)\) be found when assuming an Euclidean geometry \((G_a)\)?
What is Momentum?

Newton’s First Law of Motion

*Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.*

- It is intuitively correct.
- Experimental results do not contradict it.
- Nonetheless, I want you to think about and try and answer: What mathematical formulation exists that underpins Newton’s first law?
The Wave Equation

\[ c^2 \frac{\partial^2 \Psi}{\partial p^2} - \frac{\partial^2 \Psi}{\partial t^2} = 0 \]

- Describes waves.
- Describes continuous motion and the transportation of energy.
- Can the wave equation describe particles?
- If yes, could it be the mathematical description of Newton’s first law?
A solution of \( c^2 \frac{\partial^2 \Psi}{\partial p^2} - \frac{\partial^2 \Psi}{\partial t^2} = 0 \) in a space \([XYZ]\)

We assume the wave function \( \Psi \)

- exists in the Euclidian space \([XYZ]\)
- is simultaneously a function of time \( T(t) \) and a function of position \( P(p) \)
- is the product of the square roots of the time and position functions \( (X(p) \text{ and } Y(t)) \)

\[
\Psi = \begin{cases} 
P(p) = X^2(p) \\
T(t) = Y^2(t) \\
X(p)Y(t)
\end{cases}
\]  
(1)

We can solve \( X(p) \) and \( Y(t) \)

- by taking the second partial derivatives of \( \Psi \) with respect to \( t \) and \( p \)
- the derivatives are total as \( X(p) \) and \( Y(t) \) are independent of one another, hence

\[
\frac{c^2}{X(p)} \frac{d^2 X(p)}{dp^2} = \frac{1}{Y(t)} \frac{d^2 Y(t)}{dt^2} = -\frac{\omega^2}{4}
\]  
(2)

The solution of (2) using complex numbers is

\[
X(p) = \sqrt{A} \ e^{i\omega p/2c} 
\]  
(3)

\[
Y(t) = \sqrt{A} \ e^{i\omega t/2}
\]  
(4)

\[
t = p/c
\]  
(5)

The same mapped into a Euclidean space \([XYZ]\)

\[
X(p) = \sqrt{A} (\hat{y} \cos \omega x/2c + \hat{z} \sin \omega x/2c)
\]  
(6)

\[
Y(t) = \sqrt{A} (\hat{y} \cos \omega t/2 + \hat{z} \sin \omega t/2)
\]  
(7)

\[
t = x/c
\]  
(8)

or by squaring \( X(p) \) and \( Y(t) \)

\[
\Psi = A (\hat{y} \cos \omega x/c + \hat{z} \sin \omega x/c)
\]  
(9)

\[
\Psi = A (\hat{y} \cos \omega t + \hat{z} \sin \omega t)
\]  
(10)

\[
t = x/c
\]  
(11)
Visualisation of $\Psi$

$\Psi = \begin{cases} 
A \left( \hat{y} \cos \omega x/c + \hat{z} \sin \omega x/c \right) \\
A \left( \hat{y} \cos \omega t + \hat{z} \sin \omega t \right) 
\end{cases}$

$t = x/c$
The Wave Function as a Particle

What is needed to describe particles as a wave function?

- an aether
- multi-dimensional space
- multi-dimensional solution of the wave equation
- multi-dimensional (spatial) mathematics
- aether properties to describe the observed
I demonstrated a 3-dimensional solution for

$$\frac{c^2}{\chi(p)} \frac{d^2 \chi(p)}{dp^2} = \frac{1}{\gamma(t)} \frac{d^2 \gamma(t)}{dt^2} = -\omega^2$$

Does a solution exist in 4-dimensional space $[[X Y Z U]]$, i.e.

$$\chi(p) = \mathcal{F}_X(x, y, z, u, \omega)$$
$$\gamma(t) = \mathcal{F}_\gamma(x, y, z, t, \omega)$$
$$t = \frac{u}{c}$$
Notation for a 3D Spatial Number

Euclidian space

\[ S = [i_0 i_1 i_2] \]  \hspace{1cm} \text{(12)}

spatial number

\[ sa = a_0 i_0 + a_1 i_1 + a_2 i_2 \]  \hspace{1cm} \text{(13)}

\[ = r e^{s \theta} \]  \hspace{1cm} \text{(14)}

\[ = r e^{i_1 \theta_1} e^{i_2 \theta_2} \]  \hspace{1cm} \text{(15)}

spatial angle

\[ s \theta = s \{ \theta_1, \theta_2 \} \]  \hspace{1cm} \text{(16)}

coefficients

\[ a_0 = c \cos s \theta = \cos \theta_1 \cos \theta_2 \]  \hspace{1cm} \text{(17)}

\[ a_1 = s \cos s \theta = \sin \theta_1 \cos \theta_2 \]  \hspace{1cm} \text{(18)}

\[ a_2 = \sin s \theta = \sin \theta_2 \]  \hspace{1cm} \text{(19)}
The Algebra of Spatial Numbers

**Products of the spatial unit numbers:** The spatial-algebra defines three rules (the fourth is derived):

\[
\begin{align*}
    s i^2 & \Rightarrow -1 \quad (20a) \\
    i_h^2 & \Rightarrow -e^{s i s \{\theta_1, \theta_2, \ldots, \theta_g\}} \quad (20b) \\
    i_h^3 & \Rightarrow -i_h \quad (20c) \\
    i_h e^{s i s \{\theta_1, \theta_2, \ldots, \theta_g\}} & \Rightarrow i_h \quad (20d)
\end{align*}
\]

with \( h = g + 1 \).

**Example:** The three-dimensional spatial number

\[
\begin{align*}
a & = e^{s i s (\alpha_1, \alpha_2)} = e^{i_1 \alpha_1} e^{i_2 \alpha_2} \\
& = e^{i_1 \alpha_1} (\cos \alpha_2 + i_2 \sin \alpha_2) \\
& \text{applying eq. (20d), i.e. the fourth rule} \\
& = e^{i_1 \alpha_1} \cos \alpha_2 + i_2 \sin \alpha_2 \\
& = \cos \alpha_1 \cos \alpha_2 + i_1 \sin \alpha_1 \cos \alpha_2 + i_2 \sin \alpha_2
\end{align*}
\]
Multiplication of Spatial Numbers

The binary operation of spatial multiplication is performed by addition of the rotation angles, and is defined as:

\[ r_a \mathbf{e}^{i_s \alpha} \cdot r_b \mathbf{e}^{i_s \beta} = r_a r_b \mathbf{e}^{i_s \{\alpha_1 + \beta_1, \alpha_2 + \beta_2, \ldots, \alpha_n + \beta_n\}} \]  
(21)

The unitary operation of exponentiation is performed by multiplying each rotation angle by the exponent \( x \), and is defined as:

\[ \left( \mathbf{e}^{i_s \{\theta_1, \theta_2, \ldots, \theta_n\}} \right)^x = \mathbf{e}^{i_s \{x\theta_1, x\theta_2, \ldots, x\theta_n\}} \]  
(22)

which includes inversion when \( x = -1 \), therefore

\[ \mathbf{e}^{i_s \theta} \cdot \left( \mathbf{e}^{i_s \theta} \right)^{-1} = 1 = \left( \mathbf{e}^{i_s \theta} \right)^{-1} \cdot \mathbf{e}^{i_s \theta} \]  
(23)
Spatial Numbers form an Abelian Group

- Spatial numbers are commutative and associative under multiplication and addition

\[(s_a + s_b) + s_c = (s_b + s_c) + s_a\]
\[(s_a \times s_b) \div s_c = (s_b \div s_c) \times s_a\]

- However, the set is not distributive for multiplication over addition:

\[(s_a + s_b)s_c \neq s_a s_c + s_b s_c\]
Abelian Group

The abelian group axioms:

- **Closure**: For all \( a, b \) in \( A \), the result of the operation \( a \circ b \) is also in \( A \).

- **Associativity**: For all \( a, b \) and \( c \) in \( A \), the equation \( (a \circ b) \circ c = a \circ (b \circ c) \) holds.

- **Identity element**: There exists an element \( e \) in \( A \), such that for all elements \( a \) in \( A \), the equation \( e \circ a = a \circ e = a \) holds.

- **Inverse element**: For each \( a \) in \( A \), there exists an element \( b \) in \( A \) such that \( a \circ b = b \circ a = e \), where \( e \) is the identity element.

- **Commutativity**: For all \( a, b \) in \( A \), \( a \circ b = b \circ a \).
Spatial Numbers in Cartesian Form are Indeterminate

If $\theta_n = \pi/2$ then in cartesian form $\theta_k, 1 \leq k < n$ is indeterminate.

$$e_s i_s \{\theta_1, \pi/2\} = i_2$$  \hspace{1cm} (24)

Note that symmetries around $\pi i$ and $\pi/2$ cannot be resolved from the cartesian form

$$e_s i_s \{\theta_1, \theta_2\} = a_0 i_0 + a_1 i_1 + a_2 i_2$$  \hspace{1cm} (25)

$$= e_s i_s \{\pi + \theta_1, \pi - \theta_2\}$$  \hspace{1cm} (26)
Differentiation and Integration

Let
\[ s\phi = e^{s_i s\alpha} = e^{s_i s\{\alpha_1,\alpha_2...\alpha_n\}} \]  \hspace{1cm} (27)

then
\[ \frac{d s\phi}{d s\alpha} = s_i e^{s_i s\alpha} \quad \text{and} \quad \int s\phi \, d s\alpha = -s_i e^{s_i s\alpha} \]  \hspace{1cm} (28)

then
\[ \frac{d^2 s\phi}{d s\alpha^2} = -e^{s_i s\alpha} \quad \text{and} \quad \iint s\phi \, d s\alpha^2 = -e^{s_i s\alpha} \]  \hspace{1cm} (29)
Differentiation and Integration

Let 
\[ s\varphi = e^{s_i s\omega t} = e^{s_i \{\omega_1, \omega_2...\omega_n\}t} \]  

then 
\[ \frac{ds\varphi}{dt} = s_i s\omega e^{s_i s\omega t} \]  
and  
\[ \int s\varphi dt = -\frac{s_i e^{s_i s\omega t}}{s\omega} \]  

then 
\[ \frac{d^2 s\varphi}{dt^2} = -s\omega^2 e^{s_i s\omega t} \]  
and  
\[ \int s\varphi dt^2 = -\frac{e^{s_i s\omega t}}{s\omega^2} \]
Differentiation

Let
\[ s\varphi = e^{s\mathbf{i}s\omega t} = e^{s\mathbf{i}s\{\omega_1, \omega_2 \ldots \omega_n\}t} \] (33)

then
\[ \frac{d\, s\varphi}{dt} = s\mathbf{i}s\omega\, e^{s\mathbf{i}s\omega t} = s\mathbf{i}\sqrt{\omega_1^2 + \omega_2^2 + \ldots + \omega_n^2}\, e^{s\mathbf{i}s\omega t} \] (34)

and
\[ \frac{d^2 s\varphi}{dt^2} = -s\omega^2\, e^{s\mathbf{i}s\omega t} = -(\omega_1^2 + \omega_2^2 + \ldots + \omega_n^2)\, e^{s\mathbf{i}s\omega t} \] (35)
A solution of \( c^2 \frac{\partial^2 \Psi}{\partial p^2} - \frac{\partial^2 \Psi}{\partial t^2} = 0 \) in a space \([XYZ]\)

We assume the wave function \( \Psi \)

- exists in the Euclidian space \([XYZ]\)
- is simultaneously a function of time \( T(t) \) and a function of position \( P(p) \)
- is the product of the square roots of the time and position functions \( (X(p) \text{ and } Y(t)) \)

\[ \Psi = \begin{cases} P(p) = X^2(p) \\ T(t) = Y^2(t) \\ X(p)Y(t) \end{cases} \]  \( \text{(1)} \)

We can solve \( X(p) \) and \( Y(t) \)

- by taking the second partial derivatives of \( \Psi \) with respect to \( t \) and \( p \)
- the derivatives are total as \( X(p) \) and \( Y(t) \) are independent of one another, hence

\[ \frac{c^2}{X(p)} \frac{d^2 X(p)}{dp^2} = \frac{1}{Y(t)} \frac{d^2 Y(t)}{dt^2} = -\omega^2 \frac{\omega^2}{4} \]  \( \text{(2)} \)

which are the equations for an undamped harmonic oscillator

The solution of (2) using complex numbers is

\[ X(p) = \sqrt{\mathcal{A}} e^{i\omega p/2c} \]  \( \text{(3)} \)
\[ Y(t) = \sqrt{\mathcal{A}} e^{i\omega t/2} \]  \( \text{(4)} \)
\[ t = p/c \]  \( \text{(5)} \)

The same mapped into a Euclidian space \([XYZ]\)

\[ X(p) = \sqrt{\mathcal{A}} \left( \hat{y} \cos \omega x/2c + \hat{z} \sin \omega x/2c \right) \]  \( \text{(6)} \)
\[ Y(t) = \sqrt{\mathcal{A}} \left( \hat{y} \cos \omega t/2 + \hat{z} \sin \omega t/2 \right) \]  \( \text{(7)} \)
\[ t = x/c \]  \( \text{(8)} \)

or by squaring \( X(p) \) and \( Y(t) \)

\[ \Psi = \mathcal{A} \left( \hat{y} \cos \omega x/c + \hat{z} \sin \omega x/c \right) \]  \( \text{(9)} \)
\[ \Psi = \mathcal{A} \left( \hat{y} \cos \omega t + \hat{z} \sin \omega t \right) \]  \( \text{(10)} \)
\[ t = x/c \]  \( \text{(11)} \)
A solution of \( \frac{c^2}{p^2} \frac{\partial^2 \Psi}{\partial p^2} - \frac{\partial^2 \Psi}{\partial t^2} = 0 \) in a space \([XYZU]\)

We assume the wave function \( \Psi \)
- exists in the Euclidian space \([XYZU]\)
- is simultaneously a function of time \( T(t) \) and a function of position \( P(p) \)
- is the product of the square roots of the time and position functions \( (\mathcal{X}(p) \text{ and } \mathcal{Y}(t)) \)

\[
\Psi = \begin{cases} 
P(p) = \mathcal{X}^2(p) \\
T(t) = \mathcal{Y}^2(t) \\
\mathcal{X}(p)\mathcal{Y}(t) 
\end{cases}
\]  

(1)

We can solve \( \mathcal{X}(p) \) and \( \mathcal{Y}(t) \)
- by taking the second partial derivatives of \( \Psi \) with respect to \( t \) and \( p \)
- the derivatives are total as \( \mathcal{X}(p) \) and \( \mathcal{Y}(t) \) are independent of one another, hence

\[
\frac{c^2}{\mathcal{X}(p)} \frac{d^2 \mathcal{X}(p)}{dp^2} = \frac{1}{\mathcal{Y}(t)} \frac{d^2 \mathcal{Y}(t)}{dt^2} = -\frac{s\omega^2}{4}
\]  

(2)

which are the equations for an undamped harmonic oscillator

The solution of (2) using complex numbers is

\[
\mathcal{X}(p) = \sqrt{A} \ e^{is\omega p/2c}
\]  

(3)

\[
\mathcal{Y}(t) = \sqrt{A} \ e^{is\omega t/2}
\]  

(4)

\[
t = p/c
\]  

(5)

The same mapped into a Euclidean space \([XYZU]\)

\[
Y(t) = \sqrt{A} \left( \hat{x} \cos \omega_1 t/2 \cos \omega_2 t/2 \\
+ \hat{y} \sin \omega_1 t/2 \cos \omega_2 t/2 + \hat{z} \sin \omega_2 t/2 \right)
\]  

(36)

\[
t = u/c
\]  

(37)

or by squaring \( \mathcal{Y}(t) \)

\[
\Psi = A \left( \hat{x} \cos \omega_1 t \cos \omega_2 t \\
+ \hat{y} \sin \omega_1 t \cos \omega_2 t + \hat{z} \sin \omega_2 t \right)
\]  

(38)

\[
t = u/c
\]  

(39)
Visualisation of $\Psi$ in $[\text{XYZU}]$

$$p = \frac{2\pi c}{\omega_2}$$

$$\Psi = \left\{ \begin{array}{l}
\mathcal{A}\left(\sqrt{x^2 + y^2 \cos \omega_2 u/c} + \hat{z}\sin \omega_2 u/c\right) \\
\mathcal{A}\left(\sqrt{x^2 + y^2 \cos \omega_2 t} + \hat{z}\sin \omega_2 t\right) \\
t = u/c
\end{array} \right.$$
Let’s consider \( s\Psi(t) \) the time component of the wave function

\[
s\Psi(t) = e^{s\omega t}
\]

As per its definition it also is an eigenfunction of the linear operator \( \Lambda s\Psi = \frac{d}{dt} \) such that

\[
\Lambda s\Psi s\Psi(t) = -\lambda s\Psi(t) \quad (40)
\]

where \( \lambda = s\omega^2 \). Eigenvalues are usually associated with boundary conditions i.e. the two end points of a vibrating string.

A boundary condition for \( s\Psi(t) \) exists only if the spatial angle

\[
s\omega t = s\{\omega_1, \omega_2, \omega_3 \ldots \omega_n\} t \quad (41)
\]

is so dimensioned that it is ensured that the path followed from any point on the spherical curve, defined by \( s\Psi(t) \), always repeats itself exactly in time. Therefore any point on the spherical curve defined by \( s\Psi(t) \) can be used as a boundary. This only happens when the ratios

\[
\frac{\omega_1 : \omega_2 : \ldots : \omega_n}{\min (\omega_1, \omega_2, \ldots \omega_n)} \quad (42)
\]

are integer ratios.
Spatial Objects: Purpose and Requirements

Purpose

- Possible mathematical description of a slow moving particle that is a solution of the wave equations

Requirement

- Cartesian space.
- Space characteristics that enables the support a wave function i.e. transportivity defined as $T_0 = \frac{1}{\epsilon_0 \mu_0}$
- Independent variable for integration or differentiation i.e. time.
The solution of

\[ c^2 \frac{\partial^2 \Psi}{\partial p^2} - \frac{\partial^2 \Psi}{\partial t^2} = 0 \]

in \([XYZU]\) is fully described by following notation:

\[ s\Psi(t) = \mathcal{A} \left[_{s(\omega_1,\omega_2)}^{s(\theta_1,\theta_2)} t \right]_{u(\phi)} @p_0 + \hat{\kappa} ct \]  \hspace{1cm} (43)

where \( \cos \phi = \kappa_u \), \( \hat{\kappa} \) is a unit vector defining the direction of propagation in \([XYZU]\).
Transportivity of space: $\mathcal{T}_0 = c_0^2 = 1/\varepsilon_0 \mu_0$

Space satiation constant: $G = G/c_0^4$

Reduced Planck constant: $\hbar = \hbar/2\pi$

Energy: $E$

Momentum vector: $\vec{\rho} = \hat{\kappa} E/c_0$

Inertial mass: $m = E/c_0^2$
Spatial Objects and Inertial Mass

The object
\[
\mathcal{A}_{XYZ} \int_{s \{\omega_1, \omega_2 \} t}^u u(\phi) @ \hat{\kappa} c t
\]
has energy \( E = \hbar \sqrt{\omega_1^2 + \omega_2^2} \) thus \( \mathcal{A} = GE \)

Therefore, a stationary object in \([XYZ]\) is expressed as
\[
\mathcal{G} m_0 c_0^2_{XYZ} \int_{s \{\omega_1, \omega_2 \} t}^u u(\phi) @ \vec{\kappa}_{u_0} c t
\]
adding momentum in the \( \hat{x} \) direction to obtain
\[
\mathcal{G} m_v c_0^2_{XYZ} \int_{s \{\omega_1, \omega_2 \} t}^u u(\phi) @ (\vec{\kappa}_{x_v} + \vec{\kappa}_{u_v}) c t
\]

The velocity in \([XYZ]\) is \( v_x = c_0 \sin \phi \)

however, the momentum in \([U]\) remains unchanged, hence
\[
\vec{p}_0 = \vec{\kappa}_{u_0} m_0 c_0 = \vec{\kappa}_{u_v} m_v c_0
\]
as \( \vec{\kappa}_{u_v} = \vec{\kappa}_{u_0} \cos \phi \) and \( \cos \phi = \sqrt{1 - v^2/c_0^2} \)

the inertial mass \( m_v \) is
\[
m_v = \frac{m_0}{\sqrt{1 - v^2/c_0^2}}
\]

Newton’s first law and inertial mass explained from mathematical principles.
The Space

Spatial objects can only interact with space. Two spacial objects seem to interact with each other if and only if a spatial object modifies the space transportivity $\mathcal{T}$

$$\mathcal{T}(r) = \mathcal{T}_0 - \mathcal{D}(r)$$  \hspace{1cm} (44)

where

$$\mathcal{D}(r) = \frac{G Ec_0^2}{r} = \frac{Gm}{r}$$  \hspace{1cm} (45)
Two Objects in Space

Assume a primordial spacial object

\[ s\psi_0(t) = \frac{GM_0 c_0^2}{\text{XYZ} \cdot U} [s\{\omega_1, \omega_2\} t] \quad @ \vec{K}_u c_0 t \]

that splits, or condenses into two

\[ s\psi_0(t) \Rightarrow s\psi_1(t) \oplus s\psi_2(t) \]

The energy is

\[ E_1 = m_1 c_0^2 = m_1 (c_{10}^2 + c_{12}^2) \quad (46) \]
\[ E_2 = m_2 c_0^2 = m_2 (c_{20}^2 + c_{21}^2) \quad (47) \]

\[ c_{12}^2 = \frac{Gm_2}{r} \quad ; \quad c_{21}^2 = \frac{Gm_1}{r} \quad (48) \]

rearranging above

\[ m_1 c_{10}^2 = m_1 c_0^2 - \frac{Gm_1 m_2}{r} \quad (49) \]
\[ m_2 c_{20}^2 = m_2 c_0^2 - \frac{Gm_1 m_2}{r} \quad (50) \]
### Two Objects in Space

The energy is

\[ E_1 = m_1 c^2_0 = m_1 (c^2_{10} + c^2_{12}) \]  
\[ E_2 = m_2 c^2_0 = m_2 (c^2_{20} + c^2_{21}) \]

\[ c^2_{12} = \frac{Gm_2}{r} \quad ; \quad c^2_{21} = \frac{Gm_1}{r} \]  

Differentiating (49) and (50)

\[ F = \frac{dE'_1}{dr} = \frac{dE'_2}{dr} = \frac{Gm_1 m_2}{r^2} \]  

A profound realisation that inertial mass

\[ m'_1 = \frac{c^2_{10}}{c^2_0} m_1 \quad ; \quad m'_2 = \frac{c^2_{20}}{c^2_0} m_1 \]

decreases with increasing gravitational fields.

The total energy of a universe consisting of two objects remains constant irrespective of distance between the two objects and the local speed of light decreases

\[ c_{20} = \sqrt{c^2_0 - \frac{Gm_1}{r}} \quad \text{and} \quad c_{10} = \sqrt{c^2_0 - \frac{Gm_2}{r}} \]
Energies vs Velocities of a two body system

\[ AB = \frac{m_2 \nu c_2}{\sqrt{1 - \nu^2 / c_2^2}} \]

\[ PE = \frac{G m_1 m_2}{r} \]

\[ c_{20}^2 = c_0^2 - \frac{G m_1}{r} \]

\[ \nu_2 = c_0 \frac{c_{20}^2}{c_0^2} \sqrt{1 - \frac{c_{20}^4}{c_0^4}} \]
A spatial object with spin

$$s\Psi_0 = \frac{gE}{XYZ} \left[ s\{\omega_1 + \omega_s, \omega_2\} t \right] \hat{\kappa} c_0 t$$

with

$$E = m_0 c_0^2 + I_0 \omega_0^2$$

and when $$s\Psi_0$$ splits, or condenses, into two spatial objects $$s\Psi_1$$ and $$s\Psi_2$$ this spin is preserved as

- Spin of $$s\Psi_1$$ and $$s\Psi_2$$
- and/or $$s\Psi_1$$ and $$s\Psi_2$$ form a two body orbiting system.

For $$m_2 < m_1$$ and $$Gm_1/r << c_0^2$$ we set the orbit kinetic energy of $$m_1$$ and $$m_2$$ to have following fraction of the potential energy (CD in figure above)

$$\frac{m_2 v_2^2}{2} = \frac{Gm_1 m_2}{2(1 + x)^2 r_o} \quad (53)$$

$$\frac{m_1 v_1^2}{2} = \frac{Gm_1 m_2}{2 \frac{(1+x)^2}{x} r_o} \quad (54)$$
the orbiting velocities $v$ can be solved as:

$$v_2 = \sqrt{\frac{Gm_1}{r_o(1 + x)^2}}$$

$$v_1 = \sqrt{\frac{Gm_2x}{r_o(1 + x)^2}}$$

as $m_2 = xm_1$ thus

$$v_1 = \sqrt{\frac{Gm_1x^2}{r_o(1 + x)^2}} = xv_2$$

The centrifugal forces for both $m_1$ and $m_2$ evaluate to

$$F_{C_2} = \frac{c_2^2}{c_0^2} \frac{m_2v_2^2}{r_o}$$

$$F_{C_1} = \frac{c_1^2}{c_0^2} \frac{m_1v_1^2}{xr_o}$$

and the gravitational force on $m_2$ and $m_1$ are:

$$F_g = \frac{Gm_1m_2}{r_o^2(1 + x)^2}$$
Spacial Objects: Spin and Orbits

the effective potential of the orbit of $m_2$ and $m_1$ is

$$V_2(r_o) = \frac{c^2_{20}}{c^2_0} \frac{L^2_2}{2m_2 r^2_o} - \frac{G m_1 m_2}{r_o(1 + x)}$$

$$V_1(xr_o) = \frac{c^2_{10}}{c^2_0} \frac{L^2_1}{2m_1 x^2 r^2_o} - \frac{G m_1 m_2}{r_o(1 + x)}$$

(59)

Multiplying and dividing the second term of (59) with $c^2_0$, thus

$$V_2(r_o) = \frac{c^2_{20}}{c^2_0} \frac{L^2_2}{2m_2 r^2_o} - \frac{G m_1}{c^2_0 r_o(1 + x)} m_2 c^2_0$$

and as $m_2 c^2_{20} = m_2 c^2_0 - \frac{G m_1 m_2}{r}$

(60)

and $r = (1 + x)r_o$

$$m_2 c^2_0 = m_2 c^2_{20} + \frac{G m_1 m_2}{r_o(1 + x)}$$

(61)

the using equation (55) we rewrite

$$m_2 c^2_0 = m_2 c^2_{20} + (1 + x)m_2 v^2_2$$

(62)

and working $L_2 = m_2 v_2 r_o$ into the second term

$$m_2 c^2_0 = m_2 c^2_{20} + (1 + x) \frac{L^2_2}{m_2 r^2_o}$$

(63)

and replacing $m_2 c^2_0$ from (63) into (60) to obtain
As $x \approx 0$ thus $c \approx c_0 \approx c_{20}$ and $r \approx r_o$
and and removing subscripts (64) reduces to
\[
V(r) = \left( \frac{L^2}{2mr^2} - \frac{GMm}{r} \right) - \frac{GML^2}{mc^2r^3}
\]
which is the effective potential of orbit; first obtained by Einstein using the
Schwarzschild exact solution of the GR
field equations to describe the advance of
Mercury’s perihelion.

\[
\begin{align*}
V_2(r_o) &= \frac{c_{20}^2}{c_0^2} \frac{L_2^2}{2m_2r_o^2} - \frac{Gm_1}{c_0^2r_o(1 + x)} m_2 c_0^2 \\
&= \frac{c_{20}^2}{c_0^2} \left( \frac{L_2^2}{2m_2r_o^2} - \frac{Gm_1 m_2}{(1 + x)r_o} \right) - \frac{Gm_1 L_2^2}{m_2 c_0^2 r_o^3}
\end{align*}
\]
Two Objects in Space and Parallel Motion

Assume a primordial spacial object

\[ s\Psi_0(t) = \frac{G M_0 c_0^2}{x y z} \left[ s\{\omega_1, \omega_2\} t \right] \hat{u} (\phi) \hat{(k_x + k_u)} c_0 t \]

that splits, or condenses into two

\[ s\Psi_0(t) \Rightarrow s\Psi_1(t) \oplus s\Psi_2(t) \]

The initial velocity \( \nu = \kappa x c = c_0 \sin \phi \) is preserved. The energy is

\[ E_1 = m_1 c_0^2 = m_1 (c_{10}^2 + c_{12}^2 + \nu^2) \]  \hspace{1cm} (66)
\[ E_2 = m_2 c_0^2 = m_2 (c_{20}^2 + c_{21}^2 + \nu^2) \]  \hspace{1cm} (67)

\[ c_{12}^2 = \frac{G m_2}{r} \frac{c_0^2 - \nu^2}{c_0^2} \]  \hspace{1cm} (68)
\[ c_{21}^2 = \frac{G m_1}{r} \frac{c_0^2 - \nu^2}{c_0^2} \]  \hspace{1cm} (69)
Rearranging (66) and (67)

\[ m_1 c_{10}^2 = m_1 c_0^2 - \frac{G m_1 m_2}{r} \left( \frac{c_0^2}{c_0^2 - \nu^2} \right) - m_1 \nu^2 \]  
(70)

\[ m_2 c_{20}^2 = m_2 c_0^2 - \frac{G m_1 m_2}{r} \left( \frac{c_0^2}{c_0^2 - \nu^2} \right) - m_2 \nu^2 \]  
(71)

and differentiating \( m_1 c_{10}^2 \) and \( m_2 c_{20}^2 \) over \( r \) we obtain the gravitational force as

\[ F = \frac{d(m_1 c_{10}^2)}{dr} = \frac{d(m_2 c_{20}^2)}{dr} = \frac{G m_1 m_2}{r^2} \left( 1 - \frac{\nu^2}{c_0^2} \right) \]  
(72)

Furthermore, that the velocity vector \( \nu \) that is impressed into \( s\Psi_1 \) and \( s\Psi_2 \) is the basis of the isotropy of the observed light propagation and frame dragging.
Charge

- Elementary charge $1.60217656510^{-19}$ \text{C}

- The coulomb is a derived SI unit
  it equals one ampere-second [ $\text{C} = \text{A} \cdot \text{s}$ ]

- The ampere [A] is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per meter of length

Electrons and ions are thought as charge carriers. Hence, electric current is explained by the flow of charge carriers.
Electric Charge and Volts, Conventional View.

- A Capacitor of $C$ farad [F] electrically charged to $V$ volt [V] has an electric charge $Q = C \cdot V$ measured in coulomb [C]
- The SI derived unit farad is defined as $[F = C/V]$
- The SI derived unit volt is defined as $[V = J/C]$

An electrically charged capacitor is thought to have an excess of charge carriers on one plate and a deficit on the other
Thought Experiment

Two equivalent views of a one farad capacitor electrically charged to 10 volt. The second capacitor has the addition of two plates in the centre electrically connected.

1 farad

2 farad

5 volt
Thought Experiment

Separating an electrically charged capacitor apart does not produce a voltage between the separated internal plates.

The electric charge of a capacitor is not stored on the endplates by charge carriers, i.e. electrons or lack of electrons.

Electric energy is stored in the dielectric, a electric potential is generated.

Electric potential could be a compression/rarefaction in space.
Spatial Objects and Charge

We recall:

For the object
\[ A_{\text{XYZ}} \int \frac{1}{u(\phi)} {\hat{\kappa}} \cdot c t \]
we defined \( A = GE \).

Therefore, a stationary object in \([\text{XYZ}]\) is expressed as
\[ \frac{Gm_0c_0^2}{\text{XYZ}} \int \frac{1}{u} {\hat{\kappa}}_{u_0} \cdot c t \]
which changed the transportivity of space
\[ \mathcal{T}(r) = \mathcal{T}_0 - \frac{GEc_0^2}{r} \]

POSTULATE: The charge of particles or ions is an imaginary mass and has units quadrature kilogram \([jkg]\).

We define a new term vigour and symbolise it with \( \mathcal{V} \), it is structured, and has units joules \([J]\).

\[ \mathcal{V} = v \mathbf{e}^{\pm sj \theta} \] (73)

or
\[ \mathcal{V} = v \cos \theta \pm j v \sin \theta \] (74)

\[ = E_M \pm j E_Q \] (75)

where \( E_M \) is the mass energy and \( Q_M \) is the charge energy.
Definitions

Transportivity of space: \( T_0 = c_0^2 = 1/\epsilon_0 \mu_0 \)

Space mass-satiation constant: \( G = G/c_0^4 \)

Space charge-satiation constant: \( K = k_e/c_0^4 = 1/4\pi \epsilon_0 c_0^4 \)

Reduced Planck constant: \( \hbar = h/2\pi \)

Vigour: \( V = m_0 c_0^2 + j_0 q_0 c_0^2 \)

Momentum vector: \( \vec{\rho} = \hat{\kappa} V/c_0 \)

Inertial mass: \( m = V/c_0^2 \)
Spatial Objects and Electron Charge

\[ V = E_M \pm \mathbf{j} E_Q \]
\[ = m_0 c_0^2 \pm \mathbf{j} q_0 c_0^2 \quad (76) \]

We define the space satiation as

\[ S = G E_M + \mathbf{j} \mathcal{K} E_Q \]
\[ = G m_0 c_0^2 \pm \mathbf{j} \mathcal{K} q_0 c_0^2 \quad (77) \]

where

\[ \mathcal{K} = \frac{k_e}{c_0^4} = \frac{1}{4\pi\varepsilon_0 c_0^4} \quad (79) \]

were \( k_e = \frac{1}{4\pi\varepsilon_0} \).

A spatial object transporting a vigour \( V \) distorts the space characteristics in its vicinity such that

\[ T(r) = T_0 - D(r) \quad (80) \]

and we define \( D(r) \) as

\[ D(r) = \frac{S c_0^2}{r} = \frac{G m}{r} \pm \mathbf{j} \frac{k_e q}{r} \quad (81) \]

and \( r \) is the distance from the centre of energy of the spatial object.

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Are we limited to a mere relative reality? 40/52
Forces between Charged Massive Objects - 1

Writing the vigour

\[ \mathcal{V}_1 = (m_1 + jq_1)c_0^2 = (m_1 + jq_1)(c_{10}^2 + c_{12}^2) \]  
\[ c_{12}^2 = \frac{Gm_2}{r} + jq \frac{k_eq_2}{r} \]  
\[ \mathcal{V}_2 = (m_2 + jq_2)c_0^2 = (m_2 + jq_2)(c_{20}^2 + c_{21}^2) \]  
\[ c_{21}^2 = \frac{Gm_1}{r} + jq \frac{k_eq_1}{r} \]

and rearranging above

\[ \mathcal{V}_1 = (m_1 + jq_1)c_{10}^2 - \frac{Gm_1 m_2}{r} + jq \frac{k_eq_1 q_2}{r} - \frac{k_eq_1 m_2}{r} \]  
\[ \mathcal{V}_2 = (m_2 + jq_2)c_{20}^2 - \frac{Gm_1 m_2}{r} + jq \frac{k_eq_1 q_2}{r} - \frac{k_eq_2 m_1}{r} \]
Forces between Charged Massive Objects - 2

Differentiating (84) and (85) we obtain not only the familiar

\[ F_g = \frac{Gm_1 m_2}{r^2} \text{ and } F_c = -\frac{k_e q_1 q_2}{r^2} \]

the gravitational and charge (electrical) forces as formulated by Newton and Coulomb, remembering that \( k_e = \frac{1}{4\pi \varepsilon_0} \). In addition, we obtain a new force

\[ F_{1j} = j \frac{k_e m_1 q_2 + Gq_1 m_2}{r^2} \]
\[ F_{2j} = j \frac{k_e m_2 q_1 + Gq_2 m_1}{r^2} \]  

(86)  
(87)

A force yet to be discovered by observation.
Forces on moving charges

Force on a charge in a electric field:

\[
\vec{F}_E \equiv \lim_{q \to 0} q \frac{\sqrt{1 - (\nu \cos \alpha)^2/c^2}}{\sqrt{1 - \nu^2/c^2}} \vec{E} \quad (88)
\]

where \( \cos \alpha = \frac{\vec{\nu} \cdot \vec{E}}{\nu E} \)

The Max Planck analysis of the Kaufmann–Bucherer–Neumann experiments remains unaltered.

Faraday’s law of induction:

\[
\vec{E}_i \equiv \frac{\vec{\nu} \times \vec{B}}{\sqrt{1 - (\nu \sin \beta)^2/c^2}} \quad (89)
\]

where \( \sin \beta = \frac{\vec{\nu} \times \vec{B}}{\nu B} \)

\[
\vec{F}_B \equiv \lim_{q \to 0} q \frac{\vec{\nu} \times \vec{B}}{\sqrt{1 - \nu^2/c^2} \sqrt{1 - (\nu \sin \beta)^2/c^2}} \quad (90)
\]
Modified Lorentz Force Law

As inertial masses of moving particles is invariant to the direction of velocity, i.e. transverse and longitudinal, and as length is also invariant to velocity, the centrifugal force calculates to:

\[
\vec{F}_c = -\hat{r} \frac{mv^2}{r (1 - \nu^2/c^2)^{\frac{3}{2}}}
\]

(91)

where \(\hat{r}\) is a unit vector. The design equations of CERN stay intact as Lorentz Force Law modifies to:

\[
\vec{F} = \frac{q}{\sqrt{1 - \nu^2/c^2}} \left[ \sqrt{1 - (\nu \cos \alpha)^2/c^2} \vec{E} + \frac{\vec{v} \times \vec{B}}{\sqrt{1 - (\nu \sin \beta)^2/c^2}} \right]
\]

(92)
Reminder: Spacial Objects and Spin

A spatial object with spin

\[ s\Psi_0 = \frac{GE}{\omega_{1s} + \omega_{2s}} \int_{\text{XYZ}} s\{\omega_1 + \omega_s, \omega_2\} t \hat{\kappa} c_0 t \]

with

\[ E = m_0 c_0^2 + I_0 \omega_0^2 \]

and when \( s\Psi_0 \) splits, or condenses, into two spatial objects \( s\Psi_1 \) and \( s\Psi_2 \) this spin is preserved as

- Spin of \( s\Psi_1 \) and \( s\Psi_2 \)
- and/or \( s\Psi_1 \) and \( s\Psi_2 \) form a two body orbiting system.

For \( m_2 < m_1 \) and \( Gm_1/r \ll c_0^2 \) we set the orbit kinetic energy of \( m_1 \) and \( m_2 \) to have following fraction of the potential energy (CD in figure above)

\[ \frac{m_2 v_2^2}{2} = \frac{Gm_1 m_2}{2(1 + x)^2 r_o} \]  \( (53) \)

\[ \frac{m_1 v_1^2}{2} = \frac{Gm_1 m_2}{2 \left(\frac{1+x}{x}\right)^2 r_o} \]  \( (54) \)
Reminder: Two Objects in Space and Parallel Motion

Assume a primordial spatial object

\[ s\Psi_0(t) = \frac{GM_0c_0^2}{\sqrt{\text{XYZ}}} \int_{U(\phi)} s\{\omega_1, \omega_2\} t @ (\vec{K}_x + \vec{K}_u) c_0 t \]

that splits, or condenses into two

\[ s\Psi_0(t) \Rightarrow s\Psi_1(t) + s\Psi_2(t) \]

The initial velocity \( \nu_0 = \kappa_x c = c_0 \sin \phi \) is preserved. The energy is

\[ E_1 = m_1 c_0^2 = m_1 \left( c_{10}^2 + c_{12}^2 + \nu^2 \right) \quad (66) \]
\[ E_2 = m_2 c_0^2 = m_2 \left( c_{20}^2 + c_{21}^2 + \nu^2 \right) \quad (67) \]

\[ c_{12}^2 = \frac{Gm_2}{r} \frac{c_0^2 - \nu^2}{c_0^2} \quad (68) \]
\[ c_{21}^2 = \frac{Gm_1}{r} \frac{c_0^2 - \nu^2}{c_0^2} \quad (69) \]
Reminder: Two Objects in Space and Parallel Motion

Rearranging (66) and (67)

\[ m_1 c_{10}^2 = m_1 c_0^2 - \frac{G m_1 m_2}{r} \frac{c_0^2 - \nu^2}{c_0^2} - m_1 \nu^2 \]  
\[ m_2 c_{20}^2 = m_2 c_0^2 - \frac{G m_1 m_2}{r} \frac{c_0^2 - \nu^2}{c_0^2} - m_2 \nu^2 \]  

and differentiating \( m_1 c_{10}^2 \) and \( m_2 c_{20}^2 \) over \( r \) we obtain the gravitational force as

\[ F = \frac{d(m_1 c_{10}^2)}{dr} = \frac{d(m_2 c_{20}^2)}{dr} = \frac{G m_1 m_2}{r^2} \left( 1 - \frac{\nu^2}{c_0^2} \right) \]

Furthermore, the velocity vector \( \nu \) that is impressed into \( s \Psi_1 \) and \( s \Psi_2 \) is the basis of the isotropy of the observed light propagation and frame dragging.
Wave Function of Electron and Proton

We describe the electron as

$$\Psi_e = \gamma (m_e - j_q) c^2 \int_{XYZ} U(\varphi) @ \vec{K} \, ct$$  \hspace{1cm} (93)$$

where $m_e$ is the electron mass, $q$ the charge, the uniform translatory motion is given by $\nu = c \sin \varphi$ and as the electron was accelerated by external means, energy was added to $\Psi_e$ hence the factor

$$\gamma = \frac{1}{\sqrt{1 - \nu / c^2}}$$ \hspace{1cm} (94)$$

The proton is described by

$$\Psi_p = \gamma (m_p + j Z q) c^2 \int_{XYZ} U(\varphi) @ \vec{K} \, ct$$  \hspace{1cm} (95)$$

where $m_p$ is the mass of the atomic nucleus and $Z$ is the atomic number.
Bohr Model

We assume the electron in orbit around the proton at a distance $r$. The potential energy of an electron at a distance $r$ from a proton is

$$V_e = \left( G \frac{\gamma^2 m_e m_p}{r} + k_e \frac{\gamma^2 Z q^2}{r} \right) \frac{c^2 - \nu^2}{c^2}$$

$$\approx k_e \frac{Z q^2}{r}$$

(96)

(97)

The kinetic energy of the electron in orbit can be expressed as

$$\frac{1}{2} \gamma m_e v^2 = \frac{1}{2} k_e \frac{Z q^2}{(1 + x)^2 r}$$

(98)

where $x = m_e / m_p$ and remember $\gamma = 1 / \sqrt{1 - \nu / c^2}$ where $\nu$ is the constant velocity of the atom (proton and electron) and not to be confused with $v = r \omega$ which is the orbit velocity of the electron around the atom. At the point where

$$m_e v r = n \hbar$$

(99)

where $n$ is an integer quantity, the electron orbit becomes a standing wave (already identified as such by De Broglie). At this point the electron looses its orbiting particle nature and transforms to a new wave function where the centre of $\Psi_e$ joins up with the centre of $\Psi_p$, and the orbital kinetic energy (98) is radiated away.
Bohr Model

Using (98) and (99) we can determine the radii where standing waves of \( \Psi_e \) can form as

\[
 r = \frac{\gamma n^2 \hbar^2 (1 + x)^2}{k_e Z q^2 m_e} \tag{100}
\]

and substituting this value into \( V_e/2 \), the right part of (98), we obtain the energy radiated

\[
 E = \frac{(k_e Z q^2)^2 m_e}{\gamma 2 n^2 \hbar^2 (1 + x)^2} \tag{101}
\]

where

\[
 \mathcal{K}_0 = \frac{(k_e Z q^2)^2 m_e}{2 \hbar^2 (1 + x)^2}
\]

This discussion is not complete without a mention of gravitational effects. Defining the electron mass influenced by gravitational effects as

\[
 m_{e,g} = m_e \frac{c^2 - \frac{GM}{r}}{c^2} \tag{103}
\]

and working that result into (101) we note that the atomic clock frequency decreases in high gravitational fields.
Thus the transition frequency between two shells of an atom is

\[
 f(n_2-n_1) = \sqrt{\frac{c^2 - \nu^2}{c^2}} \left( \frac{c^2 - \frac{GM}{r}}{c^2} \right) \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) K_0 \tag{104}
\]

And the ratio of two clocks frequencies e.g. GPS

\[
 \frac{f_s}{f_e} = \sqrt{\frac{c^2 - \nu_s^2}{c^2 - \nu_e^2}} \left( \frac{c^2 - \frac{GM}{r_s}}{c^2 - \frac{GM}{r_e}} \right) \tag{105}
\]
Vigour: $\mathcal{V} = E_M + jE_Q + kE_V$

Live is intrinsic to the Universe
Are We Limited to a Mere Relative Reality?

No!

I have shown you an Ansatz towards an Absolute Reality,

and it is corroborated by experience.
DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY-CONTENT?

By A. EINSTEIN

September 27, 1905

The results of the previous investigation lead to a very interesting conclusion, which is here to be deduced.

I based that investigation on the Maxwell-Hertz equations for empty space, together with the Maxwellian expression for the electromagnetic energy of space, and in addition the principle that:

The laws by which the states of physical systems alter are independent of the alternative, to which of two systems of coordinates, in uniform motion of parallel translation relatively to each other, these alterations of state are referred (principle of relativity).

With these principles* as my basis I deduced inter alia the following result (§ 8):—

Let a system of plane waves of light, referred to the system of co-ordinates \((x, y, z)\), possess the energy \(l\); let the direction of the ray (the wave-normal) make an angle \(\phi\) with the axis of \(x\) of the system. If we introduce a new system of co-ordinates \((\xi, \eta, \zeta)\) moving in uniform parallel translation with respect to the system \((x, y, z)\), and having its origin of co-ordinates in motion along the axis of \(x\) with the velocity \(v\), then this quantity of light—measured in the system \((\xi, \eta, \zeta)\)—possesses the energy

\[
l^* = l \frac{1 - \frac{v}{c}\cos\phi}{\sqrt{1 - v^2/c^2}}
\]

where \(c\) denotes the velocity of light. We shall make use of this result in what follows.

Let there be a stationary body in the system \((x, y, z)\), and let its energy—referred to the system \((x, y, z)\) be \(E_0\). Let the energy of the body relative to the system \((\xi, \eta, \zeta)\) moving as above with the velocity \(v\), be \(H_0\).

Let this body send out, in a direction making an angle \(\phi\) with the axis of \(x\), plane waves of light, of energy \(\frac{1}{2}L\) measured relatively to \((x, y, z)\), and simultaneously an equal quantity of light in the opposite direction. Meanwhile the body remains at rest with respect to the system \((x, y, z)\). The principle of energy must apply to this process, and in fact (by the principle of relativity) with respect to both sys-

---

*The principle of the constancy of the velocity of light is of course contained in Maxwell’s equations.
tems of co-ordinates. If we call the energy of the body after the emission of light $E_1$ or $H_1$ respectively, measured relatively to the system $(x, y, z)$ or $(\xi, \eta, \zeta)$ respectively, then by employing the relation given above we obtain

$$E_0 = E_1 + \frac{1}{2} L + \frac{1}{2} L,$$

$$H_0 = H_1 + \frac{1}{2} L \frac{1 - v \cos \phi}{\sqrt{1 - v^2/c^2}} + \frac{1}{2} L \frac{1 + v \cos \phi}{\sqrt{1 - v^2/c^2}} + \frac{L}{\sqrt{1 - v^2/c^2}}.$$

By subtraction we obtain from these equations

$$H_0 - E_0 - (H_1 - E_1) = L \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}.$$

The two differences of the form $H - E$ occurring in this expression have simple physical significations. $H$ and $E$ are energy values of the same body referred to two systems of co-ordinates which are in motion relatively to each other, the body being at rest in one of the two systems (system $(x, y, z)$). Thus it is clear that the difference $H - E$ can differ from the kinetic energy $K$ of the body, with respect to the other system $(\xi, \eta, \zeta)$, only by an additive constant $C$, which depends on the choice of the arbitrary additive constants of the energies $H$ and $E$. Thus we may place

$$H_0 - E_0 = K_0 + C,$$

$$H_1 - E_1 = K_1 + C,$$

since $C$ does not change during the emission of light. So we have

$$K_0 - K_1 = L \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}. $$

The kinetic energy of the body with respect to $(\xi, \eta, \zeta)$ diminishes as a result of the emission of light, and the amount of diminution is independent of the properties of the body. Moreover, the difference $K_0 - K_1$, like the kinetic energy of the electron (§ 10), depends on the velocity.

Neglecting magnitudes of fourth and higher orders we may place

$$K_0 - K_1 = \frac{1}{2} L \frac{c^2 v^2}{c^2}.$$

From this equation it directly follows that:—

*If a body gives off the energy $L$ in the form of radiation, its mass diminishes by $L/c^2$. The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that*

The mass of a body is a measure of its energy-content; if the energy changes by $L$, the mass changes in the same sense by $L/9 \times 10^{20}$, the energy being measured in ergs, and the mass in grammes.

It is not impossible that with bodies whose energy-content is variable to a high degree (e.g. with radium salts) the theory may be successfully put to the test.

If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.
Revisiting the two 1905 special relativity papers

Energies of a two photon decay of a particle with mass $m$:

\[
\begin{align*}
E &= hf \\
\delta m &= m - \delta m \\
f_0 &= \frac{\delta m c^2}{2h} \\
E &= mc^2
\end{align*}
\]

\[
\begin{align*}
E &= \frac{mc^2}{\sqrt{1-v^2/c^2}} \\
f_0 &\sqrt{\frac{c-v}{c+v}}
\end{align*}
\]
No theory should contradict itself!

Introduce two mirrors in the moving reference system and after reflection:

\[
\begin{align*}
E &= hf \\
E &= \frac{mc^2}{\sqrt{1-v^2/c^2}}
\end{align*}
\]

\[
\begin{align*}
E &= \frac{mc^2 + \delta m v^2}{1-v^2/c^2}
\end{align*}
\]